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M-theory on a time-dependent plane-wave

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ABSTRACT: We propose a matrix model on a homogeneous plane-wave background with 20 supersymmetries. This background is anti-Mach type and is related to the time-dependent plane-wave background. We study supersymmetries of this matrix model. The vacuum energy of the abelian part is also calculated. In addition we find classical solutions such as graviton solution, fuzzy sphere and hyperboloid.

KEYWORDS: M-Theory, M(atrix) Theories, Penrose limit and pp-wave background.

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1. Introduction

Superstring theories and M-theory have supergravity theories as their low energy limits, and can be realized on various classical solutions of supergravity theories. However, it is difficult to study concretely these theories on these supergravity backgrounds because of the presence of complicated interaction terms. In a recent progress of string theories, plane-wave backgrounds have been focused upon. The type-IIB pp-wave solution found in [1] can be regarded as an approximation of the AdS geometry around a certain null geodesics [2, 3] via the Penrose limit [4]. The superstring theory on this background [5, 6] was shown to be exactly solvable in the Green-Schwarz formulation, in spite of the presence of the Ramond-Ramond fluxes. This string theory was utilized to study the AdS/CFT correspondence and revealed a stringy contribution [7] beyond the supergravity analysis.

On the other hand, the matrix model on the eleven-dimensional pp-wave background was proposed by Berenstein, Maldacena and Nastase [7], which is closely related to a supermembrane theory on this background [8, 9] via the matrix regularization [10]. This pp-wave matrix model includes mass terms and the Myers term [11], and they lead to interesting physics intrinsic to the pp-wave case. The presence of mass terms would completely lift up all flat directions at the classical level, and hence it might be expected that a single supermembrane may be stabilized. In addition, the Myers term leads to various classical solutions such as a fuzzy sphere. In particular, a fuzzy sphere solution with zero energy exists, and it can be a classical vacuum of the system, and the vacuum of plane-wave matrix model gets very enriched.

As mentioned above, superstring theories and M-theory on plane-wave backgrounds are quite interesting and so have been intensively studied. A concern in most of works is, however, restricted to time-*independent* backgrounds. It is a recent interest in superstring theories and M-theory to include the time-*dependence* in our considerations. In general, it is very difficult to treat the time-dependent backgrounds and so one can hardly study string theories and M-theory on such backgrounds. Among other time-dependent backgrounds, a time-dependent plane-wave background was considered by Papadopoulos, Russo and Tseytlin [12], and they showed that a string theory on this background is exactly solvable. This solvability comes from supersymmetries preserved as a characteristic of plane-wave backgrounds. Thus, plane-wave backgrounds seem very available to study string theories and M-theory on time-dependent backgrounds.

The main focus of this paper is to study a matrix model on an anti-Mach type solution[1] [16] and study various features of this model such as supersymmetries, vacuum structure and classical solutions. The anti-Mach type solution we will consider here is related to the time-dependent homogeneous plane-wave background [17] via a coordinate transformation. A study of the matrix model on this anti-Mach type background may be helpful to grasp part of physics on time-dependent backgrounds, though the background we consider here is not a real time-dependent background since there is a *static* coordinate chart in this background. On the other hand, this anti-Mach type background can be obtained from an M-theory Gödel universe[2] via the Penrose limit, and so we may expect that our study should be closely connected to the M-theory on the Gödel universe.

Our paper is organized as follows: In section 2, we briefly review an anti-Mach type background which leads to a time-dependent plane-wave background. In section 3, the action of a matrix model on this background is proposed. In section 4, we study the vacuum structure by studying the abelian part of this matrix model. In section 5, examples of classical solutions in this model are presented. We discuss graviton solutions in our model and find fuzzy sphere and hyperbolic type solutions. Section 6 is devoted to a conclusion and discussions. In appendix A, we discuss a matrix model on the general anti-Mach type background. Appendix B is devoted to a detailed study of the energy spectrum of a part of the system.

2. Time-dependent homogeneous plane-wave

In this section we will briefly review anti-Mach type plane-wave backgrounds, which are Cahen-Wallach spaces [19] with rotation terms. These backgrounds are classical solutions of the eleven-dimensional supergravity and are related to time-dependent homogeneous plane-

[1] M-theory Gödel universe can be constructed by lifting up a maximally supersymmetric Gödel solution in the minimal supergravity in five dimensions [13]. Also, string theories on Gödel universe and homogeneous plane-wave background are discussed in [14] and [15], respectively.

[2] The Gödel universe was originally proposed in the paper [18].

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waves via time-dependent coordinate transformations. A family of them is represented (in the Brinkmann coordinates) by

$$ds^2 = -2dx^+dx^- + \mu^2 A_{ij}x^i x^j (dx^+)^2 + 2\mu f_{ij}x^i dx^j dx^+ + (dx^i)^2\,, \tag{2.1}$$

$$F_4 = \frac{\mu}{3!}dx^+ \wedge \xi_{ijk}\, dx^{ijk}\,, \tag{2.2}$$

where A_{ij} and f_{ij} are constant symmetric and anti-symmetric matrices respectively, and ξ_{ijk} is a completely anti-symmetric third-rank tensor. The nonzero-components of the spin connection for this background are

$$\omega^{+i} = A_{ij}x^j dx^+ + f_{ij}dx^j\,, \qquad \omega^{ij} = -f_{ij}dx^+\,. \tag{2.3}$$

The metric (2.1) implicitly describes the time-dependent background if A_{ij} and f_{ij} do not commute, as mentioned in [17]. In fact, the plane-wave metric (2.1) is related to the explicitly time-dependent metric

$$ds^2 = -2dx^+dx^- + \left(\mathrm{e}^{x^+ f}A_0\mathrm{e}^{-x^+ f}\right)_{ij} z^i z^j (dx^+)^2 + (dz^i)^2\,. \tag{2.4}$$

One can show that the coordinate transformation of transverse directions

$$z^i \longrightarrow w^i = \left(\mathrm{e}^{-x^+ f}\right)_{ik} z^k\,, \tag{2.5}$$

changes the metric (2.4) to the stationary form

$$ds^2 = -2dx^+dx^- + ((A_0)_{ij} - f_{ik}f_{kj})\, w^i w^j (dx^+)^2 + (dw^i)^2 - 2w^i f_{ik}dw^k dx^+\,, \tag{2.6}$$

and thus the *time-dependent* plane-wave (2.4) can be mapped to the *stationary* plane-wave as in (2.1). We should note that the word "*time-dependent*" used here would be subtle and not be accurate since we can find a *static* coordinate chart.

In this paper, we concentrate on the special case which admits 20 supersymmetries [20], where the matrices A_{ij} and f_{ij} are given by

$$A_{22} = 4(P^2-1)\,, \qquad A_{33} = A_{44} = -1\,, \qquad f_{12} = -P\,, \tag{2.7}$$

and otherwise zero. The ξ_{ijk} is written as

$$\xi_{129} = -2\,, \qquad \xi_{349} = 2P\,, \qquad \xi_{256} = -2(1-P^2)^{1/2}\,, \quad \xi_{278} = -2(1-P^2)^{1/2}\,. \tag{2.8}$$

Notably, the above A_{ij} and f_{ij} do not commute, and so this background is related to the time-dependent background. We can derive this background from the M-theory Gödel universe by the use of the Penrose limit [4]. The M-theory Gödel universe is constructed by lifting up a maximally supersymmetric Gödel solution of the minimal five-dimensional supergravity [13] (β is an arbitrary parameter):

$$ds^2_{5\mathrm{G}} = -(dt + \beta(r_1^2 d\phi_1^2 + r_2^2 d\phi_2^2))^2 + dr_1^2 + r_1^2 d\phi_1^2 + dr_2^2 + r_2^2 d\phi_2^2\,, \tag{2.9}$$

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as the product space with the six-dimensional flat space $\mathbb{R}^6$:

$$ds^2_{\rm M} = ds^2_{\rm 5G} + dz^2 + \sum_{i=5}^{9}(dx^i)^2\,, \tag{2.10}$$

where this eleven-dimensional background is equipped with the four-form field-strength:

$$F_{r_1\phi_1 56} = F_{r_1\phi_1 78} = F_{r_1\phi_1 9z} = F_{r_2\phi_2 56} = F_{r_2\phi_2 78} = F_{r_2\phi_2 9z} = -2\beta\,. \tag{2.11}$$

This background has 20 supersymmetries (not maximal 32) in eleven dimensions.

The plane-wave solution (2.1) and (2.2) also admits 20 supersymmetries, which are inherited from that of the M-theory Gödel background (2.10), and its Killing spinor was explicitly presented in [20]. That is, this background admits extra Killing spinors satisfying

$$\begin{gathered}[\gamma_i\theta^2+6\theta\gamma_i\theta+9\theta^2\gamma_i+6\gamma_i[\theta,\phi]+18[\theta,\phi]\gamma_i-144(A_{ij}\gamma^j+f_{ij}f_{jk}\gamma^k)]\epsilon = 0\,,\\ \epsilon(x^+) = e^{\frac{x^+}{2}(\phi+\frac{1}{6}\theta)}\epsilon_0\end{gathered} \tag{2.12}$$

where ϵ_0 is a constant spinor and we have defined as

$$\theta \equiv \frac{1}{3!}\xi_{ijk}\gamma^{ijk}\,, \qquad \phi \equiv \frac{1}{2}f_{ij}\gamma^{ij}\,. \tag{2.13}$$

It is worth noting that the parameter P characterizes the null geodesic around which the Penrose limit is taken. The null geodesic is chosen to have a momentum P along the six directions transverse to the five-dimensional Gödel metric [20] (For the detail of the null geodesic, see [20]). If we consider the $P = 0$ case, then the null geodesic lies in the five-dimensional Gödel space.

A Gödel universe is deeply related to pp-wave backgrounds. The relation between the Gödel universe and pp-wave background is originally discovered in the work [21] in terms of the T-duality.

3. Matrix model on a time-dependent plane-wave

Here we shall discuss a matrix model on a homogeneous plane-wave background (anti-Mach type space) whose metric is (2.1) with (2.7) and the four-form flux (2.2) with (2.8) is equipped. By generalizing the result of the work [22], we propose an action of Discrete Light-Cone Quantized (DLCQ) M-theory as follows:

$$\begin{aligned} S = \int d\tau\,\frac{1}{2}\,\mathrm{Tr}\Big[&(D_\tau X^i)^2 + \frac{1}{2}[X^i,X^j]^2 + i\psi^T D_\tau\psi - \psi^T\gamma_i[X^i,\psi] - \\ &- 4\mu^2(1-P^2)(X^2)^2 - \mu^2(X^3)^2 - \mu^2(X^4)^2 + \\ &+ 2\mu P X^1 D_\tau X^2 - 2\mu P X^2 D_\tau X^1 + \frac{\mu}{2}i\psi^T\left(W - P\gamma^{12}\right)\psi - \\ &- \frac{4}{3}i\mu\cdot 3!\Big\{X^{[1}X^2X^{9]} - PX^{[3}X^4X^{9]} + (1-P^2)^{1/2}X^{[2}X^5X^{6]} + \\ &\qquad + (1-P^2)^{1/2}X^{[2}X^7X^{8]}\Big\}\Big]\,, \end{aligned} \tag{3.1}$$

$$W = \gamma^{129} - P\gamma^{349} + (1-P^2)^{1/2}\gamma^{256} + (1-P^2)^{1/2}\gamma^{278}\,, \tag{3.2}$$

where ψ is a 16-component SO(9) spinor and γ^i's are SO(9) gamma matrices. The symbol τ is the world-volume time and the covariant derivative D_τ is defined as $D_\tau * = \partial_\tau * - i[\omega, *]$. The gauge connection ω describes the area preserving diffeomorphism in a corresponding membrane theory[3] through the matrix regularization procedure. This matrix model (3.1) includes the Myers terms due to the presence of the constant four-form flux. Hence the fuzzy sphere solutions are expected to exist as classical solutions as in the pp-wave matrix model [7]. These solutions will be discussed later. It is also an easy task to generalize the above action to that of the matrix model on the general anti-Mach type plane-wave background (2.1) with (2.2). This generalization will be discussed in appendix A.

From now on, we will discuss supersymmetries of the matrix model on the time-dependent background. The 4 dynamical and 16 kinematical supersymmetries are preserved in this background, and so the same number of supersymmetries should be preserved in the matrix model on this background. The transformation law of 4 dynamical supersymmetries is given by

$$\begin{aligned}
\delta_\epsilon X^i &= i\psi^T \gamma^i \epsilon(\tau)\,, \qquad \delta_\epsilon \omega = i\psi^T \epsilon(\tau)\,, \qquad \epsilon(\tau) \equiv \mathrm{e}^{\frac{\mu}{6}(W+3P\gamma^{12})\tau} P_1 P_2 \epsilon_0\,, \\
\delta_\epsilon \psi &= \Bigg[D_\tau X^i \gamma_i - \frac{i}{2}[X^i, X^j]\gamma_{ij} + 2\mu P (X^1\gamma^2 - X^2\gamma^1) - \\
&\qquad - \frac{\mu}{6} X^i \gamma_i \left(W + 3P\gamma^{12}\right) - \frac{\mu}{2} X^i \left(W - P\gamma^{12}\right)\gamma_i \Bigg] \epsilon(\tau)\,,
\end{aligned} \tag{3.3}$$

where ϵ_0 is a constant spinor. We have also introduced the projection operators:

$$P_1 = \frac{1}{2}\left(\mathbb{I} - \gamma_{5678}\right), \tag{3.4}$$

$$P_2 = \frac{1}{2}\left(\mathbb{I} + c\gamma_{1956} - s\gamma_{1234}\right), \tag{3.5}$$

with $c \equiv (1 - P^2)^{1/2}$ and $s \equiv P$. The projection operators P_1 and P_2 are independent of each other and satisfy the following properties:

$$P_1^2 = P_1\,, \qquad P_2^2 = P_2\,, \qquad P_1 P_2 = P_2 P_1\,. \tag{3.6}$$

Hence a quarter of dynamical supersymmetries (i.e., 4 dynamical supersymmetries) can survive. The transformation law of 16 kinematical supersymmetries is represented by

$$\delta_\eta X^i = \delta_\eta \omega = 0\,, \qquad \delta_\eta \psi = \eta(\tau)\,, \qquad \eta(\tau) = \mathrm{e}^{-\frac{\mu}{2}(W - P\gamma^{12})\tau} \eta_0\,, \tag{3.7}$$

where η_0 is a constant spinor.

We can calculate the superalgebra of our model by using the standard Dirac bracket. Our matrix model contains complicated fluxes and the characteristic parameter P, and hence its superalgebra is expected to include rotation generators (angular momenta) and nontrivial brane charges intrinsic to our system. This interesting topic will be reported in detail in another place [23].

[3] We restrict ourselves to the closed membrane case. Hence the gauge group of the matrix model is a U(N). It is an interesting issue to study open supermembrane theories on the time-dependent background we considered here.

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4. Vacuum energy of abelian matrix model

In this section we will consider an abelian matrix model i.e., a U(1) part of U(N) matrix model proposed in section 3. In general, we do not know how to solve a full U(N) matrix model, while we can explicitly solve an abelian part of U(N) matrix model and derive the spectrum and vacuum energy. In particular, the vacuum energy of the abelian part is related to the quantum stability of the system, and hence it is quite important to study the abelian part.

First, let us consider the $P=0$ case. The abelian part of the action is given by

$$S = \frac{1}{2}\int d\tau \left[(\dot{X}^i)^2 - 4\mu^2 (X^2)^2 - \mu^2 (X^3)^2 - \mu^2 (X^4)^2 + i\psi^T\dot{\psi} + \frac{\mu}{2} i\psi^T W_0 \psi \right] \tag{4.1}$$

where we introduced the notations $W_0 \equiv \gamma_{129} + \gamma_{256} + \gamma_{278}$ and $\dot{X}^i \equiv \partial_\tau X^i$. The dynamical supersymmetry transformation is given by

$$\begin{aligned} \delta_\epsilon X^i &= i\psi^T \gamma^i \epsilon(\tau)\,, \quad \epsilon(\tau) \equiv \mathrm{e}^{\frac{\mu}{6}W_0\tau} P_1 P_2 \epsilon_0\,, \\ \delta_\epsilon \psi &= \left[\dot{X}^i\gamma_i - \frac{\mu}{6}X^i\gamma_i W_0 - \frac{\mu}{2}X^i W_0 \gamma_i \right]\epsilon(\tau)\,, \end{aligned} \tag{4.2}$$

where P_1 and P_2 are written as

$$P_1 = \frac{1}{2}(\mathbb{I} - \gamma_{5678})\,, \qquad P_2 = \frac{1}{2}(\mathbb{I} + \gamma_{1956})\,.$$

On the other hand, the kinematical supersymmetry transformation is described by

$$\delta_\eta X^i = 0\,, \qquad \delta_\eta \psi = \eta(\tau)\,, \qquad \eta(\tau) \equiv \mathrm{e}^{-\frac{\mu}{2}W_0\tau}\,. \tag{4.3}$$

Now we decompose the 16-component spinor ψ as

$$\psi = \psi^{(++)} + \psi^{(+-)} + \psi^{(-+)} + \psi^{(--)}\,, \tag{4.4}$$

where each of the spinors $\psi^{(++)}$, $\psi^{(+-)}$, $\psi^{(-+)}$ and $\psi^{(--)}$ has only 4 non-trivial components and satisfies the following two eigenvalue equations:

$$\gamma_{5678}\psi^{(\pm\bullet)} = \pm\psi^{(\pm\bullet)}\,, \qquad \gamma_{1956}\psi^{(\bullet\pm)} = \pm\psi^{(\bullet\pm)}\,. \tag{4.5}$$

Notably, the matrix W_0 commutes with γ_{5678} and γ_{1956}, and hence the eigenvalue is preserved under the action of W_0 (for example, the eigenvalue of $W_0\psi^{(++)}$ is the same as $\psi^{(++)}$). By using this decomposition, we can rewrite the fermionic action as

$$\begin{aligned} S_{\mathrm{F}} = \int d\tau \Big[& i\psi^{(++)T}\dot{\psi}^{(++)} + i\psi^{(+-)T}\dot{\psi}^{(+-)} + i\psi^{(-+)T}\dot{\psi}^{(-+)} + i\psi^{(--)T}\dot{\psi}^{(--)} + \\ & + \frac{\mu}{2} i\psi^{(++)T}\Pi\psi^{(++)} - \frac{\mu}{2} i\psi^{(+-)T}\Pi\psi^{(+-)} + \\ & + \frac{3}{2}\mu i\psi^{(-+)T}\Pi\psi^{(-+)} + \frac{\mu}{2} i\psi^{(--)T}\Pi\psi^{(--)} \Big]\,, \end{aligned} \tag{4.6}$$

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where the matrix Π is defined as $\Pi \equiv \gamma_{256}$. The vacuum energy of bosons comes from the massive directions: X^2, X^3 and X^4, and the net contribution is

$$E_{\rm B} = \frac{\mu}{2}\,(2+1+1) = +2\mu\,. \tag{4.7}$$

The vacuum energy of fermions is evaluated as (for example, following the procedure used in the work [26])

$$E_{\rm F} = -\frac{\mu}{2}(1+1+1+3) = -3\mu\,. \tag{4.8}$$

Thus, the total vacuum energy of the abelian part is

$$E_{\rm tot} = -\mu\,, \tag{4.9}$$

and it is negative.[4]

Next, we will consider the $P \neq 0$ case. The action of the abelian part is written as

$$\begin{aligned} S = \frac{1}{2}\int d\tau \Big[&(\dot{X}^i)^2 - 4\mu^2(1-P^2)(X^2)^2 - \mu^2(X^3)^2 - \mu^2(X^4)^2 + \\ &+ 2\mu P(X^1\dot{X}^2 - X^2\dot{X}^1) + i\psi^T\dot{\psi} + \frac{\mu}{2}i\psi^T\left(W - P\gamma^{12}\right)\psi\Big]\,, \end{aligned} \tag{4.10}$$

where W is defined as

$$W \equiv \gamma^{129} - s\gamma^{349} + c\gamma^{256} + c\gamma^{278}\,. \tag{4.11}$$

The transformation law of 4 dynamical supersymmetries is given by

$$\begin{aligned} \delta_\epsilon X^i &= i\psi^T\gamma^i\epsilon(\tau)\,, \quad \epsilon(\tau) \equiv {\rm e}^{\frac{\mu}{6}\left(W+3P\gamma^{12}\right)\tau}P_1P_2\epsilon_0\,, \\ \delta_\epsilon\psi &= \left[\dot{X}^i\gamma_i + 2\mu P(X^1\gamma^2 - X^2\gamma^1) - \frac{\mu}{6}X^i\gamma_i\left(W + 3P\gamma^{12}\right) - \frac{\mu}{2}X^i\left(W - P\gamma^{12}\right)\gamma_i\right]\epsilon(\tau)\,, \end{aligned} \tag{4.12}$$

where ϵ_0 is a constant spinor and the projection operators are written as

$$P_1 = \frac{1}{2}\,(\mathbb{I} - \gamma_{5678})\,, \tag{4.13}$$

$$P_2 = \frac{1}{2}\,(\mathbb{I} + c\gamma_{1956} - s\gamma_{1234})\,. \tag{4.14}$$

The transformation law of 16 Kinematical supersymmetries is given in (3.7). To begin with, we will consider the bosonic part. In the $P \neq 0$ case the analysis of X^1 and X^2 directions is nontrivial. We can find that the effect of P is to shift the origin of the oscillator X^2 and to rescale the continuous spectrum of X^1, as we will discuss in appendix B. Thus, the resulting zero-point energy of bosons is identical with that in the $P = 0$ case:

$$E_{\rm B} = 2\mu\,. \tag{4.15}$$

Next let us consider the fermionic part and focus upon the fermion mass term:

$$\frac{\mu}{2}i\psi\left(W - P\gamma^{12}\right)\psi\,. \tag{4.16}$$

[4] It is shown in [24] that the vacuum energy of abelian matrix models on less supersymmetric pp-waves may be negative. The negative vacuum energy in type-IIB strings on pp-waves is discussed in [25].

We consider decomposing the spinor ψ according to the eigenvalues of γ^{5678} and $c\gamma^{1956} + s\gamma^{12345}$ in the same way as in the $P = 0$ case. Since we can rewrite $W - P\gamma^{12}$ as

$$W - P\gamma^{12} = \gamma^{129}\left(\mathbb{I} + (\mathbb{I} - \gamma^{5678})(c\gamma^{1956} + s\gamma^{1234})\right),$$

by using $\gamma^{123456789} = \mathbb{I}$, we can easily see that these matrices commute with $W - P\gamma^{12}$. Thus, the eigenvalues of the spinor are preserved under the action of $W - P\gamma^{12}$ even in the $P \neq 0$ case. By decomposing the spinor with respect to these eigenvalues, the fermion mass term can be rewritten as

$$\frac{\mu}{2} i\psi^{(++)T}\Pi'\psi^{(++)} + \frac{\mu}{2} i\psi^{(+-)T}\Pi'\psi^{(+-)} + \frac{3}{2}\mu i\psi^{(-+)T}\Pi'\psi^{(-+)} - \frac{\mu}{2} i\psi^{(--)T}\Pi'\psi^{(--)}, \quad (4.17)$$

where $\Pi' \equiv \gamma^{129}$ and the spinors ψ^{++}, ψ^{+-}, ψ^{-+} and ψ^{--} satisfy the eigenvalue equations with respect to γ^{5678} and $c\gamma^{1956} + s\gamma^{1234}$:

$$\gamma^{5678}\psi^{\pm\bullet} = \pm\psi^{\pm\bullet}, \quad (c\gamma^{1956} + s\gamma^{1234})\psi^{\bullet\pm} = \pm\psi^{\bullet\pm}. \quad (4.18)$$

Thus the vacuum energy of fermion part is represented by

$$E_{\rm F} = -\frac{\mu}{2}(1+1+3+1) = -3\mu. \quad (4.19)$$

As a result, the net vacuum energy is

$$E_{\rm tot} = -\mu, \quad (4.20)$$

and it is the same as in the $P = 0$ case. Notably, the vacuum energy is independent of P, though the system explicitly depends on P.

We can derive the spectrum of the abelian part. It is interesting to investigate whether the resulting spectrum agrees with that in a linearized supergravity around the background considered here (as was shown in case of the maximally supersymmetric pp-wave background [27]).

5. Classical solutions

In this section we will study classical solutions of our matrix model: graviton solutions, fuzzy spheres and fuzzy hyperboloids. The graviton solutions are modified because of the extra terms with f_{ij}, and the fuzzy sphere and fuzzy hyperboloid solutions should also exist due to the presence of the Myers term. After constructing classical solutions, we consider BPS equations in our matrix model.

In order to study classical solutions we need the equations of motion. We will set $\omega = \psi = 0$ for simplicity in the following consideration if it is not especially noted. By taking a variation of the action, the equations of motion are obtained as

$$\ddot{X}^1 = -[[X^1, X^i], X^i] + 2\mu P\dot{X}^2 - 2i\mu(X^2X^9 - X^9X^2), \quad (5.1)$$

[5] The choice of this matrix is not relevant to the projection operator P_2 in (4.14).

$$
\begin{aligned}
\ddot{X}^2 &= -[[X^2, X^i], X^i] - 2\mu P\dot{X}^1 - 4\mu^2(1-P^2)X^2 - 2i\mu(X^9X^1 - X^1X^9) - \\
&\quad -2i\mu(1-P^2)^{1/2}(X^5X^6 - X^6X^5 + X^7X^8 - X^8X^7)\,, && (5.2)\\
\ddot{X}^3 &= -[[X^3, X^i], X^i] - \mu^2X^3 + 2i\mu P(X^4X^9 - X^9X^4)\,, && (5.3)\\
\ddot{X}^4 &= -[[X^4, X^i], X^i] - \mu^2X^4 + 2i\mu P(X^9X^3 - X^3X^9)\,, && (5.4)\\
\ddot{X}^5 &= -[[X^5, X^i], X^i] - 2i\mu(1-P^2)^{1/2}(X^6X^2 - X^2X^6)\,, && (5.5)\\
\ddot{X}^6 &= -[[X^6, X^i], X^i] - 2i\mu(1-P^2)^{1/2}(X^2X^5 - X^5X^2)\,, && (5.6)\\
\ddot{X}^7 &= -[[X^7, X^i], X^i] - 2i\mu(1-P^2)^{1/2}(X^8X^2 - X^2X^8)\,, && (5.7)\\
\ddot{X}^8 &= -[[X^8, X^i], X^i] - 2i\mu(1-P^2)^{1/2}(X^2X^7 - X^7X^2)\,, && (5.8)\\
\ddot{X}^9 &= -[[X^9, X^i], X^i] - 2i\mu(X^1X^2 - X^2X^1) + 2i\mu P(X^3X^4 - X^4X^3)\,. && (5.9)
\end{aligned}
$$

We can evaluate the energy of the classical solution by using the classical potential V in our model:

$$
\begin{aligned}
2V = \mathrm{Tr}\Big[&-\frac{1}{2}[X^i, X^j]^2 + 4\mu^2(1-P^2)(X^2)^2 + \mu^2(X^3)^2 + \mu^2(X^4)^2 + \\
&+\frac{4}{3}i\mu\cdot 3!\Big(X^{[1}X^2X^{9]} - PX^{[3}X^4X^{9]} + \\
&\qquad + (1-P^2)^{1/2}X^{[2}X^5X^{6]} + (1-P^2)^{1/2}X^{[2}X^7X^{8]}\Big)\Big]\,. \qquad (5.10)
\end{aligned}
$$

By using the above equations of motion and potential, we will discuss classical solutions and evaluate their classical energies.

5.1 Graviton solutions

We now consider graviton solutions. In the case of a matrix model in flat space, graviton solutions describe free particles moving with constant velocities. On the other hand, the graviton solutions in the pp-wave case are modified to be oscillators because of mass terms in the action. The graviton solution in our case is slightly different from that in usual pp-wave cases as we will explain, since there are the extra terms with f_{ij}.

Let us study graviton solutions by considering diagonal matrices. For $X^5, \ldots, X^9$, the solutions describe free particles as in flat space. For X^3 and X^4 the solutions are a couple of harmonic oscillators as in usual pp-wave cases. The remaining parts are X^1 and X^2, which couple with each other as follows:

$$
\ddot{X}^1 = 2\mu P\dot{X}^2\,, \qquad \ddot{X}^2 = -2\mu P\dot{X}^1 - 4\mu^2(1-P^2)X^2\,. \qquad (5.11)
$$

We can easily solve this system of equations and the solution is represented by

$$
\begin{aligned}
X^1 &= Px_2\sin(2\mu\tau) - P\frac{v_2}{2\mu}\cos(2\mu\tau) + \left(1-P^2\right)c_0\tau + c_0'\,, && (5.12)\\
X^2 &= x_2\cos(2\mu\tau) + \frac{v_2}{2\mu}\sin(2\mu\tau) - \frac{Pc_0}{2\mu}\,. && (5.13)
\end{aligned}
$$

The constant parameters x_2 and v_2 are the initial position and the velocity of X^2, respectively. The c_0 and c_0' are arbitrary constants related to the initial velocity and the position of X^1, respectively. It should be remarked that the X^1 direction contains trigonometrical functions in spite of the absence of the mass term, because of the contribution of f_{ij}. The X^1 behaves as a harmonic oscillator in the $P=1$ case, while as a freely moving particle when $P=0$. This result shows a specific feature of our model.

In addition, we can easily find rotating solutions as in the matrix model on the pp-wave background [7]. An example of them is

$$X^3(\tau)+iX^4(\tau)=(X^3(0)+iX^4(0))\mathrm{e}^{i\mu\tau}\,, \qquad [X^3(0),X^4(0)]=0\,, \tag{5.14}$$

and its classical energy is

$$2V=\mu^2\,\mathrm{Tr}\left[(X^3(0))^2+(X^4(0))^2\right]. \tag{5.15}$$

The classical solution found above describes motion of a D-particle in our model. So far, we have considered diagonal matrices to find classical solutions. When we consider non-diagonal matrices, we can find the expanded D-particles forming a two-dimensional object (i.e., membrane) because of the Myers terms. These objects are described by fuzzy sphere solutions and are often called giant gravitons or fuzzy membranes. These solutions will be discussed in the next subsection.

5.2 Fuzzy sphere and hyperboloid solutions

From now on, we will find solutions of the fuzzy sphere type by imposing the ansatz:

$$X^1=\alpha J^1\,, \qquad X^2=\beta J^2\,, \qquad X^9=\alpha J^3\,, \qquad \text{otherwise zero} \tag{5.16}$$

where the J^a $(a=1,2,3)$ are the SU(2) generators and satisfy the SU(2) Lie algebra $[J^a,J^b]=i\epsilon_{abc}J^c$. By putting the ansatz into the equations of motion, we obtain two conditions

$$\alpha\beta^2+\alpha^3-2\mu\alpha\beta=0\,, \quad \alpha^2\beta-\mu\alpha^2+2\mu^2(1-P^2)\beta=0\,. \tag{5.17}$$

These algebraic equations can be easily solved, and we obtain two nontrivial solutions:

$$\begin{aligned}\alpha^2 &= -2\mu^2(1-P^2)+\frac{\mu^2}{2}(1+\sqrt{1+8(1-P^2)}\,)\,,\\ \beta &= \frac{3}{2}\mu-\frac{\mu}{2}\sqrt{1+8(1-P^2)}\,,\end{aligned} \tag{5.18}$$

and

$$\begin{aligned}\alpha^2 &= -2\mu^2(1-P^2)+\frac{\mu^2}{2}(1-\sqrt{1+8(1-P^2)})\,,\\ \beta &= \frac{3}{2}\mu+\frac{\mu}{2}\sqrt{1+8(1-P^2)}\,,\end{aligned} \tag{5.19}$$

and a trivial solution $\alpha=\beta=0$. The first nontrivial solution describes a fuzzy ellipsoidal sphere and the second one represents a fuzzy hyperboloid[6] in the range $0\le P\le 1$. We

[6]The fuzzy ellipsoidal sphere and fuzzy hyperboloid solutions were constructed in the pp-wave matrix model by D. Bak [28].

will briefly discuss the range of the parameter P later. In the second case the value of α becomes purely imaginary and the compact SU(2) group is replaced with the noncompact group SO(2, 1). The energies of the fuzzy ellipsoidal sphere (5.18) and hyperboloid (5.19) are represented by

$$\begin{aligned} 2V &= \left(\alpha^2\beta^2 - \frac{4}{3}\mu\alpha^2\beta\right)\mathrm{Tr}[(J^1)^2 + (J^3)^2] + \left(\alpha^4 + 4\mu^2(1-P^2)\beta^2 - \frac{4}{3}\mu\alpha^2\beta\right)\mathrm{Tr}(J^2)^2 \\ &= -\frac{\alpha^2\beta^2}{N}, \end{aligned} \tag{5.20}$$

where we have normalized the trace for the N-dimensional representation of SU(2) generators as

$$\mathrm{Tr}(J^a J^b) = \frac{1}{N}\delta^{ab}\,. \tag{5.21}$$

The energy of the fuzzy ellipsoidal sphere (5.18) is negative while that of the fuzzy hyperboloid (5.19) is positive.

Other fuzzy solutions are given by

$$\begin{aligned} X^2 &= c\cdot\beta J^1\,, \qquad X^5 = c\cdot\alpha J^2\,, \\ X^6 &= c\cdot\alpha J^3\,, \qquad \text{otherwise zero}\,, \end{aligned} \tag{5.22}$$

and

$$\begin{aligned} X^2 &= c\cdot\beta J^1\,, \qquad X^7 = c\cdot\alpha J^2\,, \\ X^8 &= c\cdot\alpha J^3\,, \qquad \text{otherwise zero}\,, \end{aligned} \tag{5.23}$$

with $c = (1-P^2)^{1/2}$. In the same way, both cases lead to the same conditions:

$$2c^3\left(\alpha^2\beta + 2\mu^2\beta - \mu\alpha^2\right) = 0\,, \qquad c^3\alpha\left(2\mu\beta - \beta^2 - \alpha^2\right) = 0\,. \tag{5.24}$$

The solutions of the above algebraic equations are

$$\alpha^2 = -3\mu^2\,, \qquad \beta = 3\mu\,, \tag{5.25}$$

and the trivial solution $\alpha = \beta = 0$. Thus, both of the nontrivial solutions (5.22) and (5.23) with (5.25) represent fuzzy hyperboloids and have the same energy given by

$$\begin{aligned} 2V &= c^4\cdot\left(4\mu^2\beta^2 + \alpha^4 - \frac{4}{3}\mu\alpha^2\beta\right)\mathrm{Tr}(J^1)^2 + \left(\alpha^2\beta^2 - \frac{4}{3}\mu\alpha^2\beta\right)\mathrm{Tr}[(J^2)^2 + (J^3)^2] \\ &= c^4\cdot\frac{(4\mu^2\beta^2 + \alpha^4 - 4\mu\alpha^2\beta + 2\alpha^2\beta^2)}{N}, \\ &= \frac{27\mu^4 c^4}{N}\,. \end{aligned} \tag{5.26}$$

We can also apply the above consideration to the X^3, X^4 and X^9 directions by supposing the following ansatz:

$$X^3 = \alpha J^1\,, \qquad X^4 = \alpha J^2\,, \qquad X^9 = \beta J^3\,, \qquad \text{otherwise zero}\,. \tag{5.27}$$

By inserting this ansatz into the equations of motion, we can obtain the two constraint conditions:

$$\mu\alpha^2 P + \alpha^2\beta = 0\,, \qquad \alpha^3 + \alpha\beta^2 + \mu^2\alpha + 2\mu P\alpha\beta = 0\,. \tag{5.28}$$

These equations can be solved and the solution is

$$\alpha^2 = -\mu^2(1-P^2)\,, \qquad \beta = -\mu P\,. \tag{5.29}$$

This is also fuzzy hyperboloid solution, whose energy is represented by

$$\begin{aligned} 2V &= \left(\alpha^2\beta^2 + \mu^2\alpha^2 + \frac{4}{3}\mu P\alpha^2\beta\right)\mathrm{Tr}[(J^1)^2 + (J^2)^2] + \left(\alpha^4 + \frac{4}{3}\mu P\alpha^2\beta\right)\mathrm{Tr}(J^3)^2 \\ &= -\frac{\mu^4(1-P^2)^2}{N} \leq 0\,. \end{aligned} \tag{5.30}$$

The classical energy of the fuzzy hyperboloid solution is negative.

Now we will discuss the range of the parameter P. For example, if we consider the $P^2 > 1$ case in the solution (5.29), then the values of α and β give the fuzzy ellipsoidal sphere solution. If $P^2 < 1$, then the shape of the solution is hyperbolic. This result implies that the shape of the solution should be modified as the parameter P changes. It should be remarked that the parameter P might run from 0 to 1. If P is out of this range, then the bosonic mass term will be tachyonic and it seems that some trouble might be caused. However, such an issue will not be caused in the Rosen coordinates since it may be intrinsic to the system described in terms of Brinkmann coordinates. The parameter P in the Rosen coordinates is not restricted to the range $P^2 < 1$ but freely take the value. In fact, the parameter P is originally introduced as a momentum along the six directions transverse to the five dimensional Gödel metric.

We will not discuss further classical solutions, but other interesting solutions may be found in this model. It is an interesting future problem to construct such a solution. The stability of the fuzzy hyperboloids, which would be non-supersymmetric, would be also interesting to examine.[7]

5.3 BPS equations

In the previous subsection, we have presented examples of classical solutions. From now on, we will discuss the condition that all of the dynamical supersymmetries are preserved, and derive BPS equations. We give a solution of the BPS equations as an example, which is not a classical solution of the equations of motion.

Let us derive BPS equations from the supersymmetric condition: $\delta_\epsilon\psi = 0$. We also turn on the gauge connection ω in this subsection. The condition for preserving all of the dynamical supersymmetries leads to the BPS equations:

$$\begin{aligned} &[X^2, X^9] = i\frac{4}{3}\mu X^1\,, \qquad [X^9, X^1] = i\frac{4}{3}\mu X^2\,, \qquad [X^1, X^2] = i\frac{4}{3}\mu X^9\,, \\ &[X^4, X^9] = -is\cdot\frac{4}{3}\mu X^3\,, \qquad [X^9, X^3] = -is\cdot\frac{4}{3}\mu X^4\,, \qquad [X^3, X^4] = -is\cdot\frac{4}{3}\mu X^9\,, \end{aligned}$$

[7]The quantum stability of fuzzy sphere is discussed in [29].

$$
\begin{aligned}
&[X^5, X^6] = ic\cdot\frac{2}{3}\alpha\mu X^2\,, \qquad [X^6, X^2] = ic\cdot\frac{4}{3}\mu X^5\,, \qquad [X^2, X^5] = ic\cdot\frac{4}{3}\mu X^6\,, \\
&[X^7, X^8] = ic\cdot\frac{2}{3}\mu(1-\alpha)X^2\,, \qquad [X^8, X^2] = ic\cdot\frac{4}{3}\mu X^7\,, \qquad [X^2, X^7] = ic\cdot\frac{4}{3}\mu X^8\,, \\
&D_\tau X^2 = -2\mu s X^1\,, \qquad D_\tau X^1 = 2\mu s X^2\,, \qquad D_\tau X^4 = \mu X^3\,, \qquad D_\tau X^3 = -\mu X^4\,, \\
&D_\tau X^5 = D_\tau X^6 = D_\tau X^7 = D_\tau X^8 = D_\tau X^9 = 0\,,
\end{aligned}
\tag{5.31}
$$

where a constant parameter α takes the value in the range $0 < \alpha < 1$. The commutators which do not appear in the conditions (5.31) should vanish (for example, $[X^1, X^5] = 0$). We note that the equations (5.31) are not necessary but sufficient conditions for preserving all of the dynamical supersymmetries.

We can find a solution of the BPS equations (5.31) represented by

$$
\begin{aligned}
X^1 &= \left(\frac{4}{3}\mu J^2\right)\otimes\mathbb{I}\otimes\mathbb{I}\otimes\mathbb{I}\,, \qquad X^9 = \left(\frac{4}{3}\mu J^1\right)\otimes\left(-s\cdot\frac{4}{3}\mu J^1\right)\otimes\mathbb{I}\otimes\mathbb{I}\,, \\
X^2 &= \left(\frac{4}{3}\mu J^3\right)\otimes\mathbb{I}\otimes\left(c\cdot\frac{4}{3}\mu J^3\right)\otimes\left(c\cdot\frac{4}{3}\mu J^3\right), \\
X^3 &= \mathbb{I}\otimes\left(-s\cdot\frac{4}{3}\mu J^2\right)\otimes\mathbb{I}\otimes\mathbb{I}\,, \qquad X^4 = \mathbb{I}\otimes\left(-s\cdot\frac{4}{3}\mu J^3\right)\otimes\mathbb{I}\otimes\mathbb{I}\,, \\
X^5 &= \mathbb{I}\otimes\mathbb{I}\otimes\left(\frac{4}{3}\mu\cdot c\sqrt{\frac{\alpha}{2}}J^1\right)\otimes\mathbb{I}\,, \qquad X^6 = \mathbb{I}\otimes\mathbb{I}\otimes\left(\frac{4}{3}\mu\cdot c\sqrt{\frac{\alpha}{2}}J^2\right)\otimes\mathbb{I}\,, \\
X^7 &= \mathbb{I}\otimes\mathbb{I}\otimes\mathbb{I}\otimes\left(\frac{4}{3}\mu\cdot c\sqrt{\frac{1-\alpha}{2}}J^1\right), \qquad X^8 = \mathbb{I}\otimes\mathbb{I}\otimes\mathbb{I}\otimes\left(\frac{4}{3}\mu\cdot c\sqrt{\frac{1-\alpha}{2}}J^2\right), \\
\omega &= \left(2\mu\cdot sJ^1\right)\otimes\left(-\mu J^1\right)\otimes\mathbb{I}\otimes\mathbb{I}\,, \qquad D_\tau X^5 = \cdots D_\tau X^9 = \psi = 0\,.
\end{aligned}
\tag{5.32}
$$

The symbol $\otimes$ means the embedding of the SU(2) generators into the matrix, and the i-th generator belongs to the N_i-dimensional representation where $N = N_1 + N_2 + N_3 + N_4$. For example, $\left(\frac{4}{3}\mu J^3\right)\otimes\mathbb{I}\otimes\left(c\cdot\frac{4}{3}\mu J^3\right)\otimes\left(c\cdot\frac{4}{3}\mu J^3\right)$ is represented by the following matrix

$$
\left(\frac{4}{3}\mu J^3\right)\otimes\mathbb{I}\otimes\left(c\cdot\frac{4}{3}\mu J^3\right)\otimes\left(c\cdot\frac{4}{3}\mu J^3\right) = \begin{pmatrix} \frac{4}{3}\mu J^3 & & & \\ & \mathbb{I} & & \\ & & c\cdot\frac{4}{3}\mu J^3 & \\ & & & c\cdot\frac{4}{3}\mu J^3 \end{pmatrix}
\begin{array}{l} \}N_1 \\ \}N_2 \\ \}N_3 \\ \}N_4 \end{array}.
$$

The above solution (5.32) preserves 4 dynamical supersymmetries while 16 kinematical supersymmetries are broken. Hence this is a 1/5(=4/20) BPS object.[8] In our case this configuration has zero energy as in the pp-wave matrix model (i.e., $V = 0$). Therefore, this configuration can appear as the classical vacuum without any cost of energy, although this is not a classical solution of the equation of motion.

[8] In the pp-wave matrix model, such fuzzy sphere solutions preserves 16 dynamical supersymmetries and are the 1/2(=16/32) BPS objects.

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6. Conclusion and discussion

We have proposed a matrix model on a homogeneous plane-wave background which is related to a time-dependent plane-wave background. This model has 20 supersymmetries. In fact, we have explicitly constructed the transformation laws of 4 dynamical and 16 kinematical supersymmetries in this model. Then, we have shown that the vacuum energy of the abelian part is negative and it is independent of P. The effect of the parameter P is to shift the origin of the harmonic oscillator X^2 and to rescale the continuous spectrum of X^1, but it does not affect the vacuum energy. We have also constructed classical solutions: graviton solution, fuzzy (ellipsoidal) sphere and fuzzy hyperboloid. The graviton solution is slightly different from that in usual pp-wave cases because of the additional parameter P. Notably, the fuzzy ellipsoidal solution (5.16) with (5.18) and the fuzzy hyperboloid (5.27) with (5.29) have negative energies.

It would be interesting to study a supermembrane theory by taking a large-N limit of our matrix model (for the pp-wave case, see [9]). By assuming the matrix regularization [10], the supermembrane action can be formally written down as follows:

$$\begin{aligned}
S &= \int d\tau d^2\sigma w(\sigma)\frac{1}{2}\Bigg[(D_\tau X^i)^2 - \frac{1}{2}\{X^i, X^j\}^2 + i\psi^T D_\tau\psi - i\psi^T\gamma_i\{X^i,\psi\} - \\
&\qquad - 4\mu^2(1-P^2)(X^2)^2 - \mu^2(X^3)^2 - \mu^2(X^4)^2 + 2\mu P X^1 D_\tau X^2 - \\
&\qquad - 2\mu P X^2 D_\tau X^1 + \frac{\mu}{2} i\psi^T\left(W - P\gamma^{12}\right)\psi \\
&\qquad + \frac{2}{3}\mu\cdot\Bigg\{\sum_{I,J,K=1,2,9} - P\sum_{I,J.K=3,4,9} + (1-P^2)^{1/2}\sum_{I,J,K=2,5,6} + \\
&\qquad\qquad + (1-P^2)^{1/2}\sum_{I,J,K=2,7,8}\Bigg\}\epsilon_{IJK}X^K\{X^I, X^J\}\Bigg], \\
W &= \gamma^{129} - P\gamma^{349} + (1-P^2)^{1/2}\gamma^{256} + (1-P^2)^{1/2}\gamma^{278}, \qquad (6.1)
\end{aligned}$$

where $D_\tau * = \partial_\tau * + \{\omega, *\}$ is a covariant derivative and $w\{A,\, B\} \equiv \epsilon_{ab}\partial_a A\partial_b B$ is a Lie bracket. The indices, a and b, represent the spacial directions of the membrane world-volume. It is interesting to study brane charges in our model since it is known that brane charges would be modified in a pp-wave background [9, 30] and in an AdS space [31] because of the non-vanishing curvature of the background. In addition, it would be interesting to study an open supermembrane theory on the time-dependent plane-wave background as in [9, 32] because this background has quite special properties and so the classification of Dirichlet branes is also nontrivial. All of the above topics will be reported in our next paper [23].

One of the most interesting subjects is the stability of a single supermembrane on the time-dependent background. A single supermembrane in flat space is unstable as is well known [33]. This fact is deeply based on the cancellation of zero-point energies between bosons and fermions. However, it is not clear in general whether the bosonic zero-point energy cancels out the fermionic one. Remarkably, our result on the vacuum energy of the abelian part of the matrix model seems to support that the zero-point energies cannot cancel out. Moreover, the flat directions are left in the time-dependent homogeneous plane-wave in contrast to the maximally supersymmetric pp-wave case. This issue is an interesting future work.

Finally, we comment on the relation of our matrix model to the time-dependent background as a closing remark. The homogeneous plane-wave background we have considered can be mapped to the explicitly time-dependent plane-wave background via the time-dependent coordinate transformation. However, once we move to the stationary frame in which the time-dependence is not manifest, the effect of the time-dependence can be expressed by a kind of vector potential, which leads to a constant magnetic field. Thus, we might expect the relationship of our model to the noncommutative geometry or Seiberg-Witten map [34]. In addition, the plane-wave geometry considered here is closely related to the Lewis-Riesenfeld invariant theory [35] as noted in [17]. This theory leads us to the quantization of the system of time-dependent harmonic oscillators. Study of such a system would be a useful laboratory to promote our understanding of time-dependent backgrounds and to develop techniques to treat theories on them. We believe that our model should be a clue to shed light on the physics on time-dependent backgrounds.

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A. Matrix model on the general homogeneous plane-wave

Here we will discuss the matrix model on the general homogeneous plane-wave background (anti-Mach space) whose metric is (2.1) and the four-form flux (2.2) is equipped. As discussed in [20], this background admits extra Killing spinors satisfying

$$[\gamma_i\theta^2+6\theta\gamma_i\theta+9\theta^2\gamma_i+6\gamma_i[\theta,\phi]+18[\theta,\phi]\gamma_i-144(A_{ij}\gamma^j+f_{ij}f_{jk}\gamma^k)]\epsilon = 0\,, \tag{A.1}$$

$$\epsilon(x^+) = e^{\frac{x^+}{2}(\phi+\frac{1}{6}W)}\epsilon_0 \tag{A.2}$$

where ϵ_0 is a constant spinor and we have defined as

$$\theta \equiv \frac{1}{3!}\xi_{ijk}\gamma^{ijk}\,, \qquad \phi \equiv \frac{1}{2}f_{ij}\gamma^{ij}\,. \tag{A.3}$$

The matrix theory action on the background is given by

$$S = \frac{1}{2}\int d\tau\ \mathrm{Tr}\left[(D_\tau X^i)^2 + \frac{1}{2}[X^i,X^j]^2 + \mu^2 A_{ij}X^iX^j - 2\mu f_{ij}X^iD_\tau X^j + i\psi^T D_\tau\psi - \right.$$
$$\left. - \psi^T\gamma_i[X^i,\psi] + \frac{\mu}{2}i\psi^T(W+\phi)\psi + \frac{2}{3}i\mu\xi_{ijk}X^iX^jX^k\right]\,, \tag{A.4}$$

where the matrix W and the covariant derivative are defined as, respectively,

$$W \equiv -\frac{1}{2}\theta\,, \qquad D_\tau * = \partial_\tau * - i[\omega,*]\,. \tag{A.5}$$

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The action (A.4) is invariant under the dynamical supersymmetry transformations

$$\begin{aligned}\delta_\epsilon X^i &= i\psi^T\gamma^i\epsilon(\tau)\,, \qquad \delta_\epsilon\omega = i\psi^T\epsilon(\tau)\,, &(A.6)\\ \delta_\epsilon\psi &= \left[D_\tau X^i\gamma_i - 2\mu f_{ij}X^i\gamma^j - \frac{i}{2}[X^i,X^j]\gamma_{ij} - \frac{\mu}{6}X^i\gamma_i(W-3\phi) - \frac{\mu}{2}X^i(W+\phi)\gamma_i\right]\epsilon(\tau)\,, &(A.7)\end{aligned}$$

provided that

$$\begin{aligned}[\gamma_i W^2 + 6W\gamma_i W + 9W^2\gamma_i - 3\gamma_i[W,\phi] - 9[W,\phi]\gamma_i - 36(A_{ij}\gamma^j + f_{ij}f_{jk}\gamma^k)]\epsilon(\tau) &= 0\,, &(A.8)\\ \epsilon(\tau) &= e^{\frac{\mu}{6}(W-3\phi)\tau}\epsilon_0\,. &(A.9)\end{aligned}$$

Equations (A.8) and (A.9) are nothing but (A.1) and (A.2) respectively. The action (A.4) is also invariant under the kinematical supersymmetries

$$\delta_\eta X^i = \delta_\eta\omega = 0\,, \qquad \delta_\eta\psi = \eta(\tau)\,, \qquad \eta(\tau) = e^{-\frac{\mu}{2}(W+\phi)\tau}\eta_0\,, \tag{A.10}$$

where η_0 is a constant spinor.

B. Analysis of X^1 and X^2 directions in the $P \neq 0$ case

Here we will examine the X^1 and X^2 directions in the $P \neq 0$ case where the analysis of these directions are quite nontrivial and complicated due to the presence of the nonzero f_{ij}. The lagrangian of this part is given by

$$L = \frac{1}{2}\left[(\dot{X}^1)^2 + (\dot{X}^2)^2 - 4\mu^2(1-P^2)(X^2)^2 + 2\mu P(X^1\dot{X}^2 - X^2\dot{X}^1)\right]. \tag{B.1}$$

By using the standard procedure, the hamiltonian can be derived as follows:

$$H = \frac{1}{2}(\pi^1 + \mu P X^2)^2 + \frac{1}{2}(\pi^2 - \mu P X^1)^2 + 2\mu^2(1-P^2)(X^2)^2\,. \tag{B.2}$$

Here we quantize the system by imposing the commutation relation:

$$[X^i, \pi^j] = i\delta^{ij}\,,$$

and will solve the energy eigenvalue problem:

$$H\Psi = E\Psi\,.$$

When we decompose the Ψ as $\Psi = \mathrm{e}^{+i\mu P X^1 X^2}\chi$, we can obtain the equation for the χ described as

$$\begin{aligned}\tilde{H}\chi &= E\chi\,,\\ \tilde{H} &\equiv \frac{1}{2}\left(\pi^1 + 2\mu P X^2\right)^2 + \frac{1}{2}(\pi^2)^2 + 2\mu^2(1-P^2)(X^2)^2\,. &(B.3)\end{aligned}$$

The expression $\chi = \mathrm{e}^{ip_1 X^1}\phi$, where p_1 is the momentum for the X^1-direction, allows us to rewrite the hamiltonian $\tilde{H}$ as

$$\tilde{H} = \frac{1}{2}(\pi^2)^2 + \frac{1}{2}(2\mu)^2\left(X^2 + \frac{1}{2\mu}p_1 P\right)^2 + \frac{1}{2}(p_1)^2(1-P^2)\,. \tag{B.4}$$

Thus, we can see that the effect of the parameter P is to shift the origin of the harmonic oscillation of the X^2-direction and to rescale the kinetic energy (continuous spectrum) of X^1.

Finally, we would like to note that the final expression of the hamiltonian is closely related to the invariant in the context of the Lewis-Riesenfeld theory [35], which can quantize the system of time-dependent harmonic oscillators.

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